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Momentum–Impulse Balance and Parachute Inflation: Fixed-Point Drops

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I. Introduction

As documented in [1], the momentum–impulse theorem (or "MI theorem") is a very useful tool for analyzing parachute inflation, given that the duration of the net drag force generated during the inflation process is a well-documented empirical input. In particular, the MI theorem can be used to extend a well-known technique for estimating the maximum drag $F_{\max}$ to *any* type of parachute and reefing designs, and to *any* type of drop conditions. This Note aims at building on the insight gained in [1], by applying the theorem to drops of hemispherical-type canopies from fixed points rather than from aircraft. Such fixed points would not only include cranes and ceilings, but also (slow-moving) lighter-than-air aircraft and even (slow-moving) personnel parachutes. This analysis shall reveal several inflation facts that were previously unknown to the parachute engineering field.

The extended $F_{\max}$ estimator technique is based on the following formula, which uses the a priori knowledge of the opening-shock factor C_k [1–5]

$$F_{\max} \equiv (SC_D)_{\mathrm{sd}}(\tfrac{1}{2}\rho V_i^2)C_k \tag{1}$$

This expression is written in terms of the dynamic pressure sustained by the parachute–payload system at the beginning of the inflation process, most typically at the moment of the full stretching of the suspension lines, and also in terms of the parachute's drag area $(SC_D)_{sd}$, generated when the parachute is fully opened and descending in a steady manner. The use of the MI theorem allows reexpressing the C_k factor in terms of the variables that are connected to the parachute–payload trajectory during inflation, namely [1]

$$C_k = \frac{2}{R_m n_{\mathrm{fill}}^{\mathrm{gen}}}\Gamma \tag{2}$$

The nondimensional number R_m is the so-called inverse mass ratio defined as [3,5]

$$R_m = \frac{\rho (SC_D)_{\mathrm{sd}}^{3/2}}{m} \tag{3}$$

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On the other hand, the "generalized" filling time $n_{\mathrm{fill}}^{\mathrm{gen}}$ is defined as

$$n_{\mathrm{fill}}^{\mathrm{gen}} \equiv \frac{V_i(t_f - t_i)}{(SC_D)_{\mathrm{sd}}^{1/2}} I_F^{\mathrm{if}} = n_{\mathrm{fill}} \frac{D_0}{(SC_D)_{\mathrm{sd}}^{1/2}} I_F^{\mathrm{if}} \tag{4}$$

This expression involves the "standard" filling time [2–4], a nondimensional measure of the inflation time given by $n_{\mathrm{fill}} \equiv (t_f - t_i)V_i/D_0$. Here D_0 is the so-called *nominal* canopy diameter, defined from the total canopy surface area S_0 as $D_0 = (4S_0/\pi)^{1/2}$ for hemispherical-type canopies. On the other hand, Eq. (2) involves the so-called "drag integral" I_F^{if} and the "MGI integral" Γ, which are defined formally as

$$I_F^{\mathrm{if}} \equiv \int_i^f \frac{|F_D(t)|\,\mathrm{d}t}{F_{\max}(t_f - t_i)} \tag{5}$$

$$\Gamma \equiv \frac{-(V_f - V_i) + \int_i^f g\cos\theta(t)\,\mathrm{d}t}{V_i} \tag{6}$$

In these equations the variables $V(t_i) \equiv V_i$ and $V(t_f) \equiv V_f$ refer to the parachute–payload tangential speeds at the beginning and end of the inflation process, respectively. V_i and V_f are always positive while $F_D(t) < 0$ and $F_{\max} > 0$, a convention defined for studying gravity-powered inflation sequences taking place either horizontally or in a generally downward direction. In practical terms, t_i refers to the moment of suspension line stretch or first "gulp"; and t_f marks, for parachute systems undergoing much deceleration during inflation (i.e., at large R_m), the moment when the canopy has inflated to its steady-descent diameter for the first time. In the case of systems that are accelerating or moving at constant speeds (i.e., low R_m), t_f typically marks the time of maximum canopy extension: a time that usually follows first time extension to steady-state diameter [1,2]. The interval t_f–t_i thus defined allows its determination from video of the opening process. On the other hand, the drag integral I_F^{if} contains the information related to the normalized but explicit shape of the F_D-vs-t curve, and is a measurement of the area under that curve. This feature encapsulates the specifics of drag evolution and actually becomes a dominant feature in the large-R_m and -n_{fill} limits, as will be discussed further below. Reference [1] discusses the values of the drag integral, estimated from current experimental data in the low- and large-R_m regimes, and suggests $I_F^{\mathrm{if}} \sim \frac{1}{2} \pm 50\%$ at high mass ratios and $I_F^{\mathrm{if}} \sim 1/5 \pm 30\%$ at low mass ratios.

The discussion of [1] shows that Eqs. (1), (2), and (6) allow useful simplifications when considering whether the inflation sequence occurs along the horizontal only, where $\Gamma = 1 - V_f/V_i$, or along the vertical where $\Gamma = 1 - V_f/V_i + (g/V_i)(t_f - t_i)$. In the case of purely vertical trajectories such a simplification amounts to

$$F_{\max} = \left(\frac{1}{2}\rho V_i^2\right)(SC_D)_{\mathrm{sd}}\left[\frac{2}{R_m n_{\mathrm{fill}}^{\mathrm{gen}}}\right]\left(1 - \frac{V_f}{V_i} + \frac{gD_0}{V_i^2}n_{\mathrm{fill}}\right) \tag{7}$$

a result that is not only valid for any values of the mass ratio R_m, but also one that can be used for any parachute and reefing type, and for any type of parachute drops performed from a nonmoving or slowly moving platform. Note that the three terms within the rightmost set of parentheses were obtained from the Γ factor in (6), and represent the momentum change experienced by the parachute payload (per unit initial speed and mass) as well as the impulse supplied by gravity (i.e., the term proportional to the gravitational acceleration constant g). Usually a small contribution for human-sized parachutes carrying human weights, the gravitational impulse can become a major contributor to inflation dynamics for very large parachutes and heavy payloads, and/or at large mass ratios and long filling times. The latter limit will be explored in more details in the sections below.

II. Drops from Fixed Points

Although not as prevalent as parachute drops from aircraft, fixed-point parachute drops offer a low-cost alternative, especially for inflation studies that require a large number of repeat launches, or, as with the case of drops performed indoors, for studies that demand a well-characterized air column into which the parachute must fall. Another significant advantage of this kind of drop is that it yields the *simplest preinflation payload fall dynamics possible*, and thus a straightforward estimation of the initial fall rate V_i, a crucial input of Eq. (7). Finally, this test drop technique offers, *with the same parachute and payload, and with similar speed evolution ranges*, the possibility of generating parachute wakes during inflation that are characterized by pure decelerations, pure accelerations, or a mix of both. This is an interesting point given that typical aircraft drops offer mainly decelerating trajectories. In other words, fixed-point drops allow studying the effects of wake aerodynamics during inflation, in a manner that downplays specific dependences on payload geometry and weight, and specific parachute shape and porosity. As discussed elsewhere [6], deceleration/acceleration profiles determine wake shape evolution and are therefore a crucial element for calculating $F_{\max}$.

So far, three main launch strategies have been considered in fixed-point drop experiments of the recent past [7–11]:

1) Bag deployed (BD): where the falling payload is pulling, out of a bag attached to the fixed point, the stowed suspension lines (first) and packed parachute (second).

2) Sleeve deployed (SD): where the falling payload is pulling, out of a sleeve attached to the fixed point, the parachute in an already unstowed and unfolded configuration.

3) Freely hanging (FH): where the parachute and lines are hanging from the fixed point, in a completely unstowed and unfolded state (the parachute being initially attached to the ceiling at its apex).

Of these three fixed-point methods, SD-based drops are those that yield the more consistent openings on a drop-to-drop basis, given that the initial motion of the parachute canopy inside the sleeve allows a small amount of internal pressurization before leaving the sleeve, thus guaranteeing a well-defined (and opened) canopy mouth area of the beginning of inflation [9].

Note also that in the case of freely hanging drops, the parachute–payload fall rate at the beginning of the inflation process is simply $V_i = 0$. But this very simplicity prevents the use of Eqs. (1–7) altogether, and will necessitate an alternative formulation of the MI theorem, as will be discussed in a later section. In contrast, both bag- and sleeve-deployed drops do allow the use of Eqs. (1–7), given that $V_i \neq 0$ in these cases.

Most importantly, fixed-point drops allow useful (and realistic) estimates of V_i if the fall rate of the payload and deploying parachute *is dominated by gravity*; indeed, using constant-acceleration kinematics one gets

$$V_i \sim \sqrt{2gL} \tag{8}$$

where

$$L = L_{\text{susplines}} + \frac{D_{\text{constr}}}{2} \qquad \text{(BD)} \tag{9}$$

or

$$L = L_{\text{sleeve}} \qquad \text{(SD)} \tag{10}$$

Here D_{constr} is the so-called "constructed diameter" [3], which is defined as twice the distance between the canopy apex and skirt (i.e., suspension line attachments); on the other hand $L_{\text{susplines}}$ and L_{sleeve} correspond to the length of the suspension lines and of the deployment sleeve, respectively. Note that L_{sleeve} can be of arbitrary length, although in the past investigators have used $L_{\text{sleeve}} \sim D_{\text{constr}}$ [9]. Note also that in the case of the BD expression, there is the assumption that the canopy apex is tied directly to the bag/ceiling via break chord, in contrast to being tied to a lanyard of arbitrary length, which would add an extra term in (9). Again, the assumption of gravity domination that is behind Eqs. (8–10) means that the friction between the sleeve/bag and canopy in SD/BD drops must be small, and that payload drag must also be small.

A final but important ingredient arises when considering fixed-point drops performed in the large-R_m regime, where peak drag is generated early in the inflation process and where a great deal of deceleration also occurs early. In this case parachute–payload fall rate V_f can be approximated by the steady-descent fall rate of the system, namely

$$V_f \sim \sqrt{\frac{2W}{\rho (SC_D)_{\text{sd}}}} \tag{11}$$

This result is all the more relevant given the fact that most fixed-point drops are slow-speed drops, that is, where $V(t) \sim V_f$ in order of magnitude.

III. Maximum Drag from Bag- or Sleeve-Deployed Drops

Using (8–11) in (7) thus produces a large R_m estimate of maximum drag, a result that can be used in most drops discussed in the literature, for example in [7–10]:

$$F_{\max} = [g\rho L (SC_D)_{\text{sd}}] \frac{\sqrt{(SC_D)_{\text{sd}}}}{D_0} \left[\frac{2}{R_m n_{\text{fill}} I_F^{\text{if}}}\right] \left(1 - \sqrt{\frac{1}{R_m} \frac{\sqrt{(SC_D)_{\text{sd}}}}{L}} + \frac{n_{\text{fill}} D_0}{2L}\right) \tag{12}$$

This result shows that the force scale is set by an air mass factor $\rho L (SC_D)_{\text{sd}}$, a mass that may be larger or smaller (depending on L) than the typical mass $\rho (SC_D)_{\text{sd}}^{3/2}$ coaccelerating or decelerating with the parachute–payload system. Note also that the $n_{\text{fill}} D_0 / 2L$-term, the gravitational impulse term, is no longer proportional to g, a direct result of having V_i^2 being proportional to g. The double square-root term in Eq. (12) represents the V_f / V_i ratio, a ratio that is shown more clearly in (7). Its value is controlled by the parachute steady-descent drag area and by the length of the deployment sleeve. In cases where a very long sleeve is used, one has $V_i > V_f$, which results mostly into a parachute–payload system sustaining a deceleration that is similar qualitatively to drops from fast aircraft. More interesting is the case of very short sleeves where $V_i < V_f$ (with $V_i \neq 0$). Here the system undergoes either a simple acceleration to V_f, or an acceleration to a value greater than V_f, followed by a deceleration to V_f, a motion profile that is rarely encountered in typical aircraft drops.

Note that when $V_i < V_f$, the sum $(1 - V_f / V_i)$ in Eq. (7) may become negative. Because the value of $F_{\max}$ must remain positive by definition, the gravitational term $n_{\text{fill}} D_0 / 2L$ must always be greater than the sum $1 - V_f / V_i$ in such a case. In other words, Eqs. (7) and (12) establish a lower bound for the filling time. Such a lower bound is a general characteristic of Eq. (7), which in dimensional terms means that one must always have $(t_f - t_i) > (V_i / g) |(1 - V_f / V_i)|$ along purely vertical trajectories. This bound is trivially small for cases where $V_i > V_f$, or when $V_i < V_f$, with V_i being similar to V_f. On the other hand it becomes large, and thus more interesting from a practical point of view, when $V_i \ll V_f$ (with V_i being small). The filling time being bounded from below is actually a general property of the MI theorem whenever it is used for nonhorizontal inflation trajectories. This bound can be traced to the basic fact that the total energy being dissipated by parachute drag is at least equal to the gravitational energy gained during inflation (with the rest of the energy budget coming from the kinetic energy available to the parachute–payload system at the beginning of inflation, and the kinetic energy that may have been gained during inflation).

Equation (12) shows also that the details of the drag evolution, that is, $F_D(t)$ as "encapsulated" in I_F^{if}, may not be as important as the actual value of the filling time n_{fill}, a parameter that depends not only on parachute and reefing design, but also on the actual state of the parachute mouth at the very beginning of the inflation sequence (more discussion on the filling time can be found in [1]). But such relative importance changes drastically when evaluating (12) in the limits of *large*-R_m and *large*-n_{fill}:

$$\lim_{\substack{R_m \to \text{large} \\ n_{\text{fill}} \to \infty}} F_{\max} = \frac{W}{I_F^{\text{if}}} \tag{13}$$

This simple result follows from the $n_{\text{fill}} D_0/2L$ term in (12) becoming dominant. In other words, a lot more momentum is being gained by the system via gravity and lost through drag. Note that in the real world, mass ratio and filling time are never infinitely large; but with typical values such as $n_{\text{fill}} \sim 10$ and $R_m \sim 1$–10 [2,3], this term becomes large enough to make (13) practically testable when $L \sim D_0$. Note also that such a limit would never arise from an inflation event taking place along a horizontal trajectory.

Equation (13) is interesting for other reasons: 1) $F_{\max}$ has become independent of inflation time, unlike the usual inflation sequence where $F_{\max} \sim 1/n_{\text{fill}}$ [Eq. (1)]; 2) to the extent that current experimental data determines the drag integral at $I_F^{\text{if}} \sim 1/2 \pm 50\%$ [1], one has $F_{\max} \sim 2W$; and 3) the value of $F_{\max}$ is no longer dependent explicitly on canopy dimensions and porosity; total weight and force evolution shape is all that matters. But even more interesting is the fact that this limit holds also for the case of freely hanging drops as discussed next.

IV. Maximum Drag from Freely Hanging Drops

Despite its simplicity from a hardware and procedural point of view, FH-type drops present the difficulty associated with having $V_i = 0$, thus preventing the use of Eqs. (1–7). An alternative formulation of the MI theorem is now developed, where the final speed V_f is used instead to set the distance and temporal scales in the definitions of the nondimensional opening shock factor and filling time:

$$C_k^{\text{FH}} \equiv \frac{F_{\max}}{(SC_D)_{\text{sd}}(\frac{1}{2}\rho V_f^2)} \tag{14}$$

$$n_{\text{fill}}^{\text{genFH}} \equiv \frac{V_f(t_f - t_i)}{(SC_D)_{\text{sd}}^{1/2}} I_F^{\text{if}} = n_{\text{fill}}^{\text{FH}} \frac{D_0}{(SC_D)_{\text{sd}}^{1/2}} I_F^{\text{if}} \tag{15}$$

Now Eq. (15) involves a "modified" standard filling time; namely, one based on V_f: $n_{\text{fill}}^{\text{FH}} \equiv (t_f - t_i)V_f/D_0$. On the other hand, the drag integral I_F^{if} appearing here is still given by Eq. (5). Using the MI theorem for purely vertical drops, namely [1]

$$mV_f - mV_i = mV_f = \int_i^f F_D(t)\,\mathrm{d}t + W(t_f - t_i) \tag{16}$$

together with (14) and (15) one obtains

$$C_k^{\text{FH}} = \frac{2}{R_m n_{\text{fill}}^{\text{genFH}}} \left[\frac{-V_f + g(t_f - t_i)}{V_f}\right] \tag{17}$$

that is, a form that is very similar to (2) [along with using (6)]. On the other hand, rearranging (14) and (17) together with (11) gives the following result, which is valid only at large values of mass ratio [because of the use of (11)]:

$$F_{\max} = W \frac{\sqrt{(SC_D)_{\text{sd}}}}{D_0} \left[\frac{2}{R_m n_{\text{fill}}^{\text{FH}} I_F^{\text{if}}}\right] \left(n_{\text{fill}}^{\text{FH}} \frac{R_m}{2} \frac{D_0}{\sqrt{(SC_D)_{\text{sd}}}} - 1\right) \tag{18}$$

Once again, it is to be noted that the gravitational impulse term, that is the term in $n_{\text{fill}}^{\text{FH}} R_m$, is again g independent as with Eq. (12). Comparing further (18) with (12), one finds the force scale of $F_{\max}$ being set by total weight W instead of by air mass. But, to the extent that FH drops are performed at similar R_m values and filling times (i. e., $n_{\text{fill}} \sim n_{\text{fill}}^{\text{FH}}$), both equations should yield similar values of $F_{\max}$. This is seen quite clearly in the limit of large R_m and large n_{fill}, where it can be shown that (18) converges into (13) very much as (12) did; namely, as $F_{\max} \to W/I_F^{\text{if}}$. Again, this result arises from the gravitational impulse becoming the dominating factor.

V. Conclusions

This Note has shown that the use of fixed-point parachute drops adds interesting acceleration and deceleration profiles for the study of parachute inflation, profiles that are rarely available with standard drops from aircraft. Moreover, using the MI theorem for their analysis turned out an interesting lower bound for the filling time. Finally, it is interesting to note that in the large-R_m and large-n_{fill} limits, the *load factor* $F_{\max}/W$ becomes amazingly simple by virtue of (13), that is, $F_{\max}/W \to 1/I_F^{\text{if}}$ for both FH and BD/SD drops. In other words, expressing the maximum force in g's becomes solely a statement about the area under the normalized $F_D(t)$ vs t curve. Such a statement is strikingly simple given the complexity of the inflation process. Similar easy-to-remember expressions shall be found with other parachute systems as well [12–14].

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